

Introducing the density of states per unit energy interval,

$$v(\epsilon_F) = V p_F^2 / 2\pi^2 v_F$$

and using formulas (2) and (3), we express F in the form

$$F = \frac{1}{2(4\pi)^2} \frac{N_0}{M s^2 v(\epsilon_F)}$$

In this formula it is convenient to go over from the speed of sound s to the Debye temperature $\omega_D = \pi s/a$, where a is the lattice constant equal to $(V/N_0)^{1/3}$; then the expression for T_k can be re-written in the form

$$T_k = 1.14 \omega_D \exp \left\{ - \left(\frac{N_0}{V} \right)^{1/3} \frac{N_0}{32 v(\epsilon_F) M \omega_D^2} \right\}^{-1} \quad (4)$$

whence

$$\frac{\partial T_k}{\partial P} = T_k \kappa \left\{ \gamma_p \left(1 - \frac{2}{F} \right) + \frac{\gamma_e}{F} + \frac{2}{3F} \right\}, \quad (5)$$

where we have introduced the notation

$$\kappa \gamma_p = \frac{1}{\omega_D} \frac{\partial \omega_D}{\partial P}, \quad \kappa \gamma_e = - \frac{1}{v(\epsilon_F)} \frac{\partial}{\partial P} v(\epsilon_F), \quad \kappa = - \frac{1}{V} \frac{\partial}{\partial P} V. \quad (5')$$

The quantities γ_p and γ_e are the Grüneisen constants for the lattice and for the electrons and κ is the compressibility.

Assuming that formula (5) is correct not only for a quadratic dispersion law, but also for an arbitrary law $\epsilon(p)$, we compare the theory and experiment.

The experimental values of the quantities γ_p and γ_e and κ known at present are presented in the Table for a number of metals. Using these values it is easy to calculate $T_{k,P}$ from formula (5). It can be seen from the Table that the experimentally observed values of $T_{k,P}$ and those calculated according to formula (5) are generally in good agreement.

In those instances in which the Grüneisen constant γ_e for electrons is not known, we assumed that $\gamma_e = 1.5$ (this corresponds approximately to the average of the known values). The average value of γ_e yields the correct sign of the derivative of the temperature of the superconducting transition with respect to the pressure.

The calculated values of $T_{k,P}$ for all metals differ nevertheless from the experimentally found values of $T_{k,P}$. In principle this could be related to the fact that the constant γ_e has not been determined sufficiently well, for an error in γ_e (or γ_p) of 10–20 per cent leads to an error of 50–100 per cent in the calculated value of $T_{k,P}$ because of the presence of the large parameter F^{-1} .

Analyzing the data presented in the Table (the seventh and eighth column) one can assume that the Grüneisen constant γ_e for the metals In, Hg, Zn, Cd, Zr, Mo, and La $_{\beta}$ can differ from $\gamma_e = 1.5$ by no more than $\pm(0.1-0.2)$.

3. The value of the critical magnetic field in the superconductor at temperatures T much smaller than T_k is connected with the superconducting transition temperature by the relation

$$H_k(0) = 1.75 [4\pi v(\epsilon_F) / V]^{1/2} T_k. \quad (6)$$

Differentiating this expression with respect to the pressure, one can express the derivative $H_{k,P} \equiv \partial H_k(0) / \partial P$, as was done for $T_{k,P}$, in terms of the Grüneisen constants γ_e and γ_p and the compressibility κ :

$$\frac{\partial H_k}{\partial P} = \kappa H_k \left\{ \left(1 - \frac{2}{F} \right) \left(\gamma_p - \frac{1}{2} \gamma_e \right) + \frac{2}{3} \frac{1}{F} + \frac{1}{2} \right\}. \quad (7)$$

On the basis of this formula one can calculate the magnitude and the sign of $H_{k,P}$ for various metals and compare it with the experimentally found values of $H_{k,P}$ (see the Table). As can be seen from the Table, the agreement for $H_{k,P}$, just as for $T_{k,P}$, between the calculated and experimentally determined values is not bad.

The values of $T_{k,P}$ and $H_{k,P}$ are determined for 19 pure metals. For Ga, Re, Ru, Th, and Ti γ_e and γ_p have not been determined and therefore a comparison with the theory is difficult.

For Tl, if we assume $\gamma_e \approx 1.5$, then according to the theory $T_{k,P} = -5 \times 10^{-5}$ deg/atm; the experimentally determined value $T_{k,P}$ is -1.4×10^{-5} deg/atm at 20,000–28,000 atm. The thallium anomalies in the pressure range up to 6000 atm are apparently connected with features of the energy spectrum of the conduction electrons.^[14,15]

The authors thank B. G. Lazarev for a discussion of the work.

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